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Sensitivity analysis of the pond heating and temperature regulation (PHATR) model

Jonathan Lamoureux [a], Terrence R. Tiersch [b], Steven G. Hall [a,*]

[a] *Department of Biological and Agricultural Engineering, Louisiana Agricultural Experiment Station, Louisiana State University Agricultural Center, 143 Doran, Baton Rouge, LA 70803, USA*
[b] *Aquaculture Research Station, Louisiana Agricultural Experiment Station, Louisiana State University Agricultural Center, Baton Rouge, LA 70803, USA*

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Abstract

The PHATR model was designed to determine: (1) the temperature and (2) the size of the energy transfer mechanisms for research-sized (400 m^3) outdoor aquaculture ponds in Louisiana. Because various environmental parameters affect the model's output, a sensitivity analysis was used to identify which factors had a greater affect on the pond temperature. Four environmental parameters (air temperature, solar radiation, wind speed and flow rate of water used to control the pond temperature) were varied one at a time. The temperature for 1 m^3 ponds was modeled for 48 h. The output (pond temperature) for each trial model run was compared to the output of a standard model run. The model's sensitivity to air temperature varied linearly ($r = 0.999$) from 0.10 to 0.35 °C/°C. The model's sensitivity to solar radiation ranged from 0.04 to 0.14 °C/W. The model's sensitivity to wind speed (dependent on wind speed) ranged from −0.003 to −1.64 °C/(m/s). The model's sensitivity to the flow of warm water (dependent on the flow of warm water) ranged from 158,074 to 620,845 °C/(m^3/s)/m^3. The model's sensitivity to the flow of cool water (dependent on the flow of cool water) ranged from 46,375 to 844,873 °C/(m^3/s)/m^3. For all cases, time was found to have an effect on the model's sensitivity. Ultimately, this data is useful for the design and management of outdoor earthen aquaculture ponds in the Southeastern United States or similar areas.

Keywords: Aquaculture; Energy balance modeling; Sensitivity analysis; Temperature control

1. Introduction

Energy transfer mechanisms in nature have been studied previously (Anonymous, 1992; Holman, 1997). Energy transfer components, for example, longwave radiation exchange (Bliss, 1961; Swinbank, 1963), evaporation from bodies of water (Anonymous, 1952), surface convection for flat surfaces (McAdams, 1942; Watmuff et al., 1977), soil heat transfer (Van Wijk and de Vries, 1966) and solar radiation (Threlkeld and Jordan, 1958) have also been investigated. A heat transfer model, based on these studies, was used to estimate energy gains and losses in aquacultural ponds (Lamoureux et al., 2006).

* Corresponding author. Tel.: +1 225 578 1049; fax: +1 225 578 3492.
E-mail address: shall@bae.lsu.edu (S.G. Hall).

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The pond heating and temperature regulation model (PHATR) is a computer model developed to: (1) predict the temperature of outdoor earthen aquaculture ponds and (2) to estimate the size of the heat transfer mechanisms responsible for pond temperature changes (Lamoureux, 2003). Specifically designed for the warmwater ponds at the Louisiana State Agricultural Center Aquaculture Research Station (ARS), PHATR numerically solves Eq. (1), the mathematical representation of an energy balance for a pond:

$$\rho c_p V \frac{dT}{dt} = q_{solar} - q_{pond} + q_{sky} \pm q_{conv} - q_{evap} + q_{well} - q_{out} \quad (1)$$

where ρ is the density of water (kg/m^3), c_p the specific heat of water (J/kg°C), V the volume of the pond (m^3), T the pond temperature (°C), t the time (s), q_{solar} the solar radiation captured by the pond (W), q_{pond} the long wave radiation emitted by the pond (W), q_{sky} the long wave sky radiation captured by the pond (W), q_{conv} the heat transferred at the water surface (W), q_{evap} the rate of energy lost through water evaporation (W), q_{well} the rate of energy gained from the warm well water pumped into the pond (W) and q_{out} is the rate of energy lost to the water discharged from the pond (W). Losses due to seepage were ignored because of the practical difficulties involved in measuring the mass flow rate, although these losses could be significant under some field conditions. Other sources of energy such as the respiration of fish or decomposing bacteria on the pond bottom, work done by a continuously operating aerator or heat transfer with the soil were ignored (Lamoureux et al., 2006). Each energy vector was calculated based on equations from the literature (Lamoureux et al., 2006).

As a design tool, PHATR can help size temperature control technologies and systems for outdoor aquaculture facilities. As a management tool, PHATR can assist farmers and researchers regulate energy entering their ponds, usually in the form of inexpensive geothermal or industrial warm water.

PHATR's predictions rely on input data, describing the weather, the flow of warm water into the pond and the physical characteristics of the pond itself. The input data can be actual or simulated. How the input data affects the model's output can be studied with a sensitivity analysis.

A sensitivity analysis was used to reveal which changes in the input caused greater impacts on the output. This was especially important when considering uncertainties or errors inherent in the measured or predictive input data. The sensitivity analysis also identified certain scenarios where varying a certain input variable had a counter-intuitive effect on the pond temperature.

2. Materials and methods

For this sensitivity analysis, a standard model run, with fixed input variables, was compared to trial model runs. For each trial, only one input variable in the standard model run was changed by a predetermined increment. By comparing changes in the input to changes in the output, the model's sensitivity was calculated.

2.1. Description of the standard model run

The standard model run consisted of running PHATR for 2 hypothetical days (172,800 s) for a hypothetical pond with dimensions 1 m × 1 m × 1 m. Edge effects were ignored to simplify the analysis. In a large pond, edge effects would be much less than in a very small pond, so this assumption should be reasonable in many cases. Because many channel catfish ponds are 1.25–1.50 m deep, and because certain pond energy transfer mechanisms were affected by the size of the pond surface, regardless of depth, results cannot necessarily be used for scaling purposes. For the standard model run, there was no warm or cool water flowing in the pond. The pond temperature at the start of the 2-day period was 20 °C.

The weather during these 2 hypothetical days was clear (no clouds) with an average air temperature of 20 °C. The air temperature varied sinusoidally. The daytime high, 25 °C, occurred at 14:00 (2 p.m.) and the daytime low, 15 °C, occurred at 2:00 (2 a.m.). Solar radiation varied sinusoidally between 6:00 and 18:00 (6 a.m. and 6 p.m.), the maximum (1355 W/m^2) occurring at 12:00 (noon). (The value 1355 W/m^2 is the incident solar radiation above the atmosphere – Holman, 1997 – and was used in this analysis as an absolute maximum). Between 18:00 and 6:00, the solar radiation was set to 0 W/m^2. The relative humidity was set constant at 90%. There was no wind (wind speed = 0 m/s). All weather variables for the standard are shown in Appendix A.

2.2. Description of the trial model runs

For each model run, one variable was changed while keeping all other weather conditions at their hypothetical standard value. Four variables were changed:

- The average air temperature was set at either 0, 10, 15, 25, 30 or 40 °C (6 runs). Diurnal fluctuations were modeled as a sinusoidal function with an amplitude of 5 °C. The daytime high occurred at 14:00 and the night time low occurred at 2:00.
- The solar radiation was varied by −25, −50, −75 and −100% (4 runs) of the standard solar radiation value modeled by the equation in Appendix A.
- The wind speed was set to 0.5, 1, 2, 5, 10, 20 and 50 m/s (7 runs).
- The flow rate was varied on a "per volume" scale. For instance, a flow rate of 200 l/min in a 400 m^3 pond (the average volume for warm water ponds at the LSU ARS) corresponds to a flow rate of 8.33×10^{-6} m^3/s per unit of pond volume. The flow rate was set to 2.08, 4.17, 8.33, 16.7, 25.0 and 33.3×10^{-6} m^3/s per unit of pond volume (m^3). In every case, the flow rate was set to 0 m^3/s during the daytime. The temperature of the inlet water was set at 36 °C (warm water trials) and 15 °C (cool water trials) (12 runs).

All model runs, including the standard model run, used a time step of 10 min (600 s) and simulated a period of 2 days.

3. Results

3.1. Temperature changes

Model run results for the pond temperature were compared to the standard model run over the 2-day period (Fig. 1).

Lowering the average air temperature resulted in relatively lower pond temperatures at the end of the 2-day period (see Fig. 1a). For instance, the pond temperature, after being exposed to an average air temperature of 0 °C for 48 h, was 26.7 and 6.3 °C below the standard model run pond temperature (33.0 °C). Alternately, increasing the average air temperature raised the pond temperature. The pond temperature, after being exposed to 40 °C air for 48 h, was 40.5 and 7.5 °C above the standard model run pond temperature. As time progressed, the absolute difference between the standard temperature and the resulting trial temperature increased. For instance, the temperature difference between the standard model run and the 40 °C trial model run increased from 2.0 °C after 12 h to 3.9 °C after 24 h, to 5.8 °C after 36 h, to 7.5 °C after 48 h. These changes over time were found to be linear ($r > 0.999$), as will be discussed latter.

Decreasing the amount of incident solar radiation decreased the pond temperature (Fig. 1b). This made sense since solar energy was found to be the major energy transfer mechanism during the day, accounting for as much as 55% of all energy vectors (Lamoureux, 2003). After 48 h, the pond temperature increased for all trials except when there was no solar radiation. In this case, the pond temperature steadily decreased almost linearly ($r = -0.955$) by 1.46 °C over 2 days.

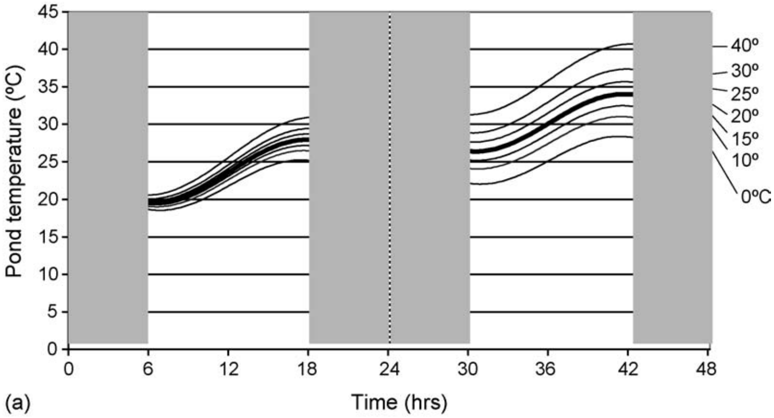
Increasing the wind speed had different effects on surface convection and evaporation, depending on the time of day (Fig. 1c). At noon on Day 2, increasing the wind speed from 0 to 5 m/s caused the pond temperature to decrease from 30.1 to 25.4 °C, mainly because the rate of energy lost by evaporation increased from 46 to 110 W. Energy losses due to evaporation decreased when the wind speed was increased beyond 5 m/s because the vapor pressure of the water, now cooler because of convection, decreased. For a wind speed of 50 m/s, there was no noontime evaporation because the water temperature was so low that the water vapor pressure (2.51 kPa) was smaller than the air vapor pressure (2.87 kPa).

Surface convection removed heat from the pond for wind speeds under 10 m/s at noon on Day 2. Because the pond temperature for a wind speed of 10 m/s, 23.3 °C, was below the air temperature, 23.5 °C, the air "warmed" the pond. By increasing the wind speed from 10 to 50 m/s, convection increased from 6 to 336 W but the pond temperature decreased from 23.3 to 21.3 °C. This was because increasing the wind speed also increased nighttime evaporation (from 401 to 761 W at midnight, on Day 2) and caused the pond temperature to fall. Note the pond vapor pressure was greater than the cooler nighttime air, making evaporation possible.

Increasing the wind speed beyond 20 m/s had an additional effect: the pond temperature peaked earlier in the day. Because there was no evaporation, air convection became an un-checked heat transfer mechanism causing the pond temperature to rise more quickly. Between 8:00 and 10:00, the pond temperature changed by 1.6 °C for a wind speed of 20 m/s. For the same time period but for a wind speed of 50 m/s, the pond temperature changed by 2.3 °C.

As was seen, increasing the wind speed did not necessarily increase the importance of convection or evaporation because both convection and evaporation were also affected by their respective gradients, the true driving force behind these energy transfer mechanisms. In the case of convection, the temperature gradient between the air and the water surface drove this transport phenomenon. In the case of evaporation, the vapor pressure gradient between the water surface and the air is the driving force. Because increasing the wind speed changed the temperature of the water as well as the transport coefficient, the gradients for heat and mass transfer also changed, affecting the amount of energy gained or lost through convection and evaporation.

Increasing the flow of warm water (36 °C) increased the rate the pond warmed between 0 and 6 a.m. on Day 1 (Fig. 1d). The warm water was shut off at 6:00 and the solar radiation was responsible for heating the ponds until 18:00. In Fig. 1d, there is a discontinuity in all the curves (except for the standard curve) at 6:00, reflecting the change in



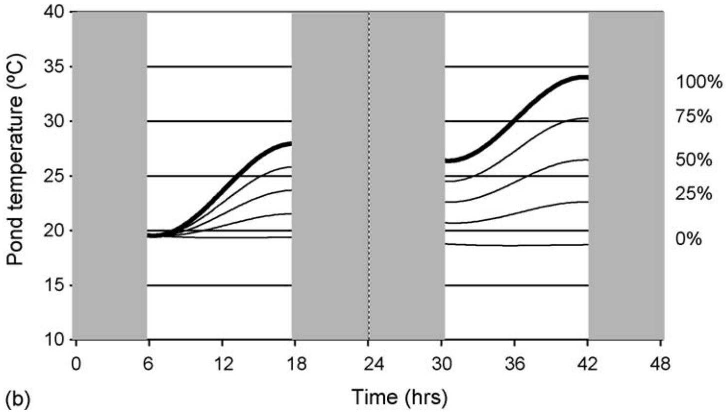


Fig. 1. Five parameters (one for each graph) were varied during the PHATR model sensitivity analysis. In each graph, each curve represented a model run where a parameter was varied. The air temperature variations (a) were 0, 10, 15, 20, 25, 30 and 40 °C. Solar radiation (b) were varied by 0, 25, 50, 75 and 100% of the values set by the standard model run. The wind speed (c) was set to 0, 0.5, 1, 2, 5, 10, 20 and 50 m/s. The flow rate for both warm 36 °C water (d) and cool 15 °C water (e) were 0, 2.1, 4.2, 8.4, 16.7, 25.0 and 33.3 × 10^{-6} (m^3/s)/m^3. The bold line in each graph is the standard model run. Each simulation was done for 48 h, with nighttime periods shaded. For graphs (d) and (e), no water was flowing during the day (unshaded area).

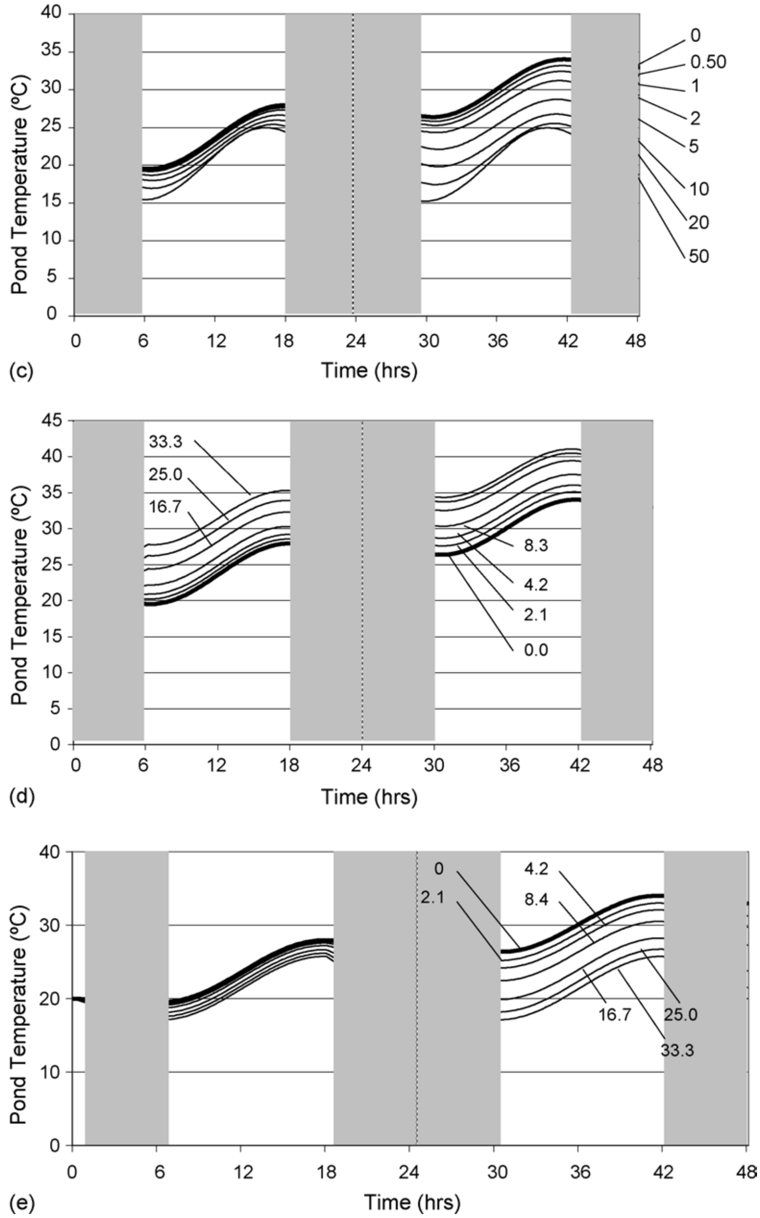


Fig. 1. (*Continued*).

heating method (from warm water heating to solar heating). During the first night, the pond temperature changed at a rate between 0.0015 °C/h (flow rate = 2.08×10^{-6} (m^3/s)/m^3) and 0.082 °C/h (flow rate = 8.33×10^{-6} (m^3/s)/m^3). These small changes in temperature over 12 h shows how during the first complete night (between 18:00 on Day 1 and 6:00 on Day 2), the energy gained by the pond through incoming warm water was balanced out by the energy lost through other energy transfer mechanisms, as well as the energy lost to the water leaving the pond (see Fig. 2). Therefore, increasing the flow rate of warm water, in this case, did very little in changing the temperature.

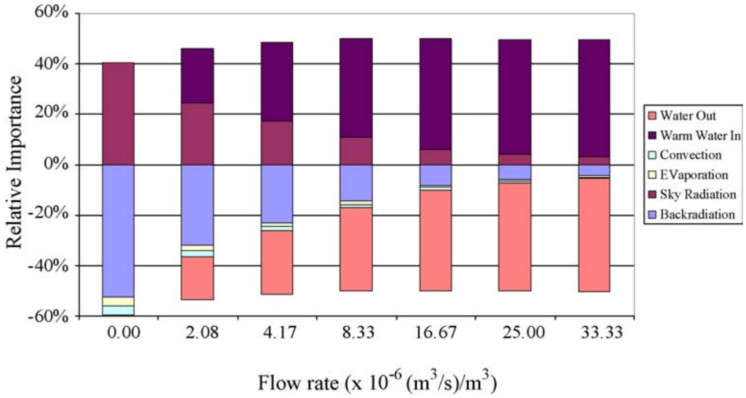


Fig. 2. Each bar shows the relative importance of each energy transfer vector for a specific flow rate at 24:00 (midnight on Day 1). As flow increased, the relative importance of long wave radiation diminished from 93 to 8% (pond and sky radiation combined). The importance of evaporation and convection also decreased from 7 to 1% (convection and evaporation combined) as the flow of warm water increased. The importance of warm inlet water and the discharged water increased as flow increased from 0 to 91%.

Increasing the flow of cool water (15 °C) to the pond decreased the pond temperature with respect to the standard model run (see Fig. 1e). The pond temperature after 48 h for the standard was 33.0 °C while the temperature of the pond after 48 h for a flow of 33.3×10^{-6} (m^3/s)/m^3 was 20.0 °C, a difference of 13.0 °C.

Depending on the flow rate, the cool water removed the energy gained by the pond during the day from solar radiation. The nighttime low, when the flow equaled 25.0×10^{-6} (m^3/s)/m^3, was 17.6 °C on the first night and 18.2 °C on the second night an increase in 0.6 °C. The nighttime low, when the flow equaled 33.3×10^{-6} (m^3/s)/m^3, was 17.1 °C on the first night and 17.1 °C on the second night. The maximum flow of cold water was needed to remove the solar energy gained during the course of the day.

The rate at which the temperature decreased in the ponds during the night was also dependent on the flow rate. For the first night, the lowest flow regime (flow = 2.08×10^{-6} m^3/s) caused the pond to cool at a rate of 0.21 °C/h whereas for the highest flow regime (flow = 33.3×10^{-6} (m^3/s)/m^3), the rate at which the pond cooled was 0.70 °C/h.

3.2. Sensitivity

Normally, the sensitivity of a model can be calculated using the following using the following equation:

$$S = \frac{\Delta\text{output}}{\Delta\text{input}} = \frac{T_{\text{trial}} - T_{\text{std}}}{P_{\text{trial}} - P_{\text{std}}} \tag{2}$$

where S is the sensitivity (units of output versus units of input), T (the output) is the trial or standard model run pond temperature (°C) and P (the input) is the trial or standard model run parameter (units to be specified). However, this equation implies that the model's sensitivity is constant, which was not the case in this study. Changes in the environmental parameters – the model's inputs – did not result in linear changes in the pond temperature – model's output (Fig. 3). To account for the sensitivity's variability, the sensitivity was redefined as the slope (or first derivative) of a curve fitted to a Δoutput versus Δinput plot:

$$S = \lim_{\Delta\ \text{input} \to 0} \left(\frac{\Delta\text{output}}{\Delta\text{input}} \right) = \frac{\mathrm{d(output)}}{\mathrm{d(input)}} = f(\text{input}) \tag{3}$$

where f is a function to be determined using experimental data.

To determine the model's sensitivity, changes in output (pond temperature) were plotted against changes in input (environmental parameters), shown in Fig. 3. A curve was statistically fitted and its equation determined using Curve Expert (Hyams, 2001). The derivative of the curve's equation was then calculated, yielding the model's sensitivity as a function of the input environmental parameter.

Changes in output were determined at 12, 24, 36 and 48 h into the model runs, yielding four curves per varied parameter, one for each time. This was done to see how the model's sensitivity changed with time.

The PHATR model's sensitivity was also plotted with respect to the model inputs (Fig. 4).

Changes in pond temperature varied linearly to changes in air temperature (Table 1 and Fig. 3a). A linear regression was applied to each curve with the correlation coefficients and the slopes shown in Table 1. All curves had a regression coefficient, r, of 0.999. As time increased from 12 to 48 h, the model's sensitivity (slope) also increased from 0.097 to 0.48 °C/°C.

The sensitivity varied linearly with time ($r = 0.999$), according to the following equation:

$$S = 0.00695t + 0.0178 \quad (4)$$

where S is the model's sensitivity to air temperature and time ((°C/°C)/h), and t is time (h).

This information is useful when estimating the effects of measurement errors or poor data on the model's output. For instance, suppose an error of 5 °C was present in the recorded air temperature data, due to some technical malfunction. The error was present for 30 h. The corresponding error in the PHATR model's output would be

$$0.00695\,^{\circ}\mathrm{C}/^{\circ}\mathrm{C\,h} \times 30\,\mathrm{h} + 0.0178\,^{\circ}\mathrm{C}/^{\circ}\mathrm{C}$$
$$= 0.2285\,^{\circ}\mathrm{C}/^{\circ}\mathrm{C}$$

An input error of 5 °C translated into an output error of 1.1425 °C.

Changes in pond temperature varied linearly to changes in solar radiation ($r > 0.999$ for all cases).

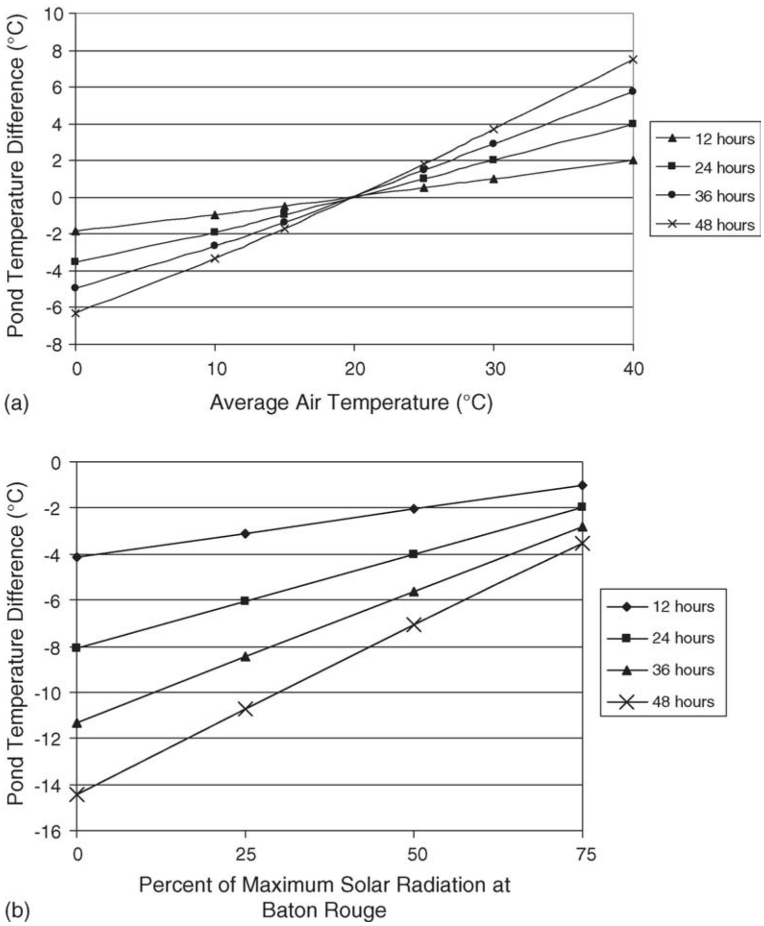


Fig. 3. Relative changes in the model output (pond temperature) were plotted against (a) air temperature (°C), (b) solar radiation (%), (c) wind speed (m/s) and the flow rate of (d) warm and (e) cool water ((m^3/s)/m^3). Because time affected the model output, to properly study the effects of each varied parameter, time had to be fixed. For each curve, time is fixed at 12, 24, 36 or 48 h.

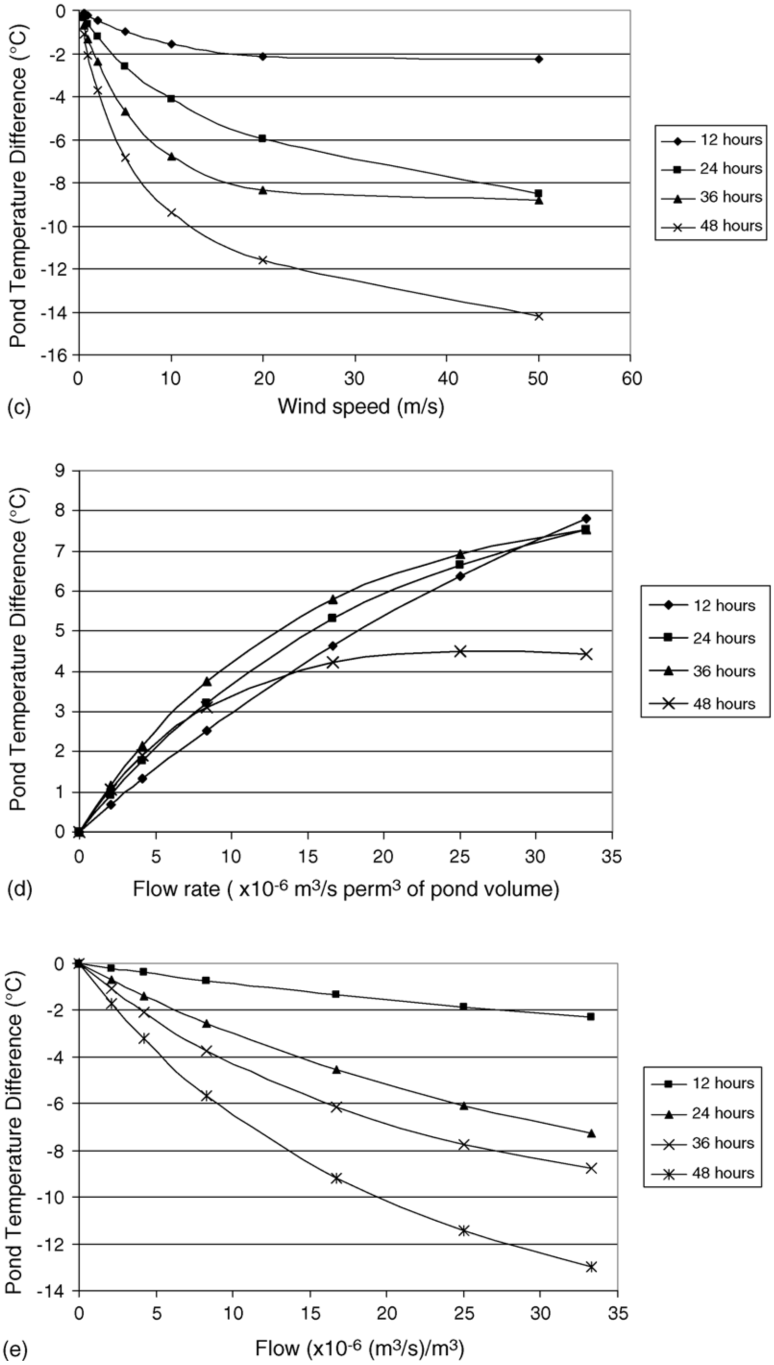


Fig. 3. (*Continued*).

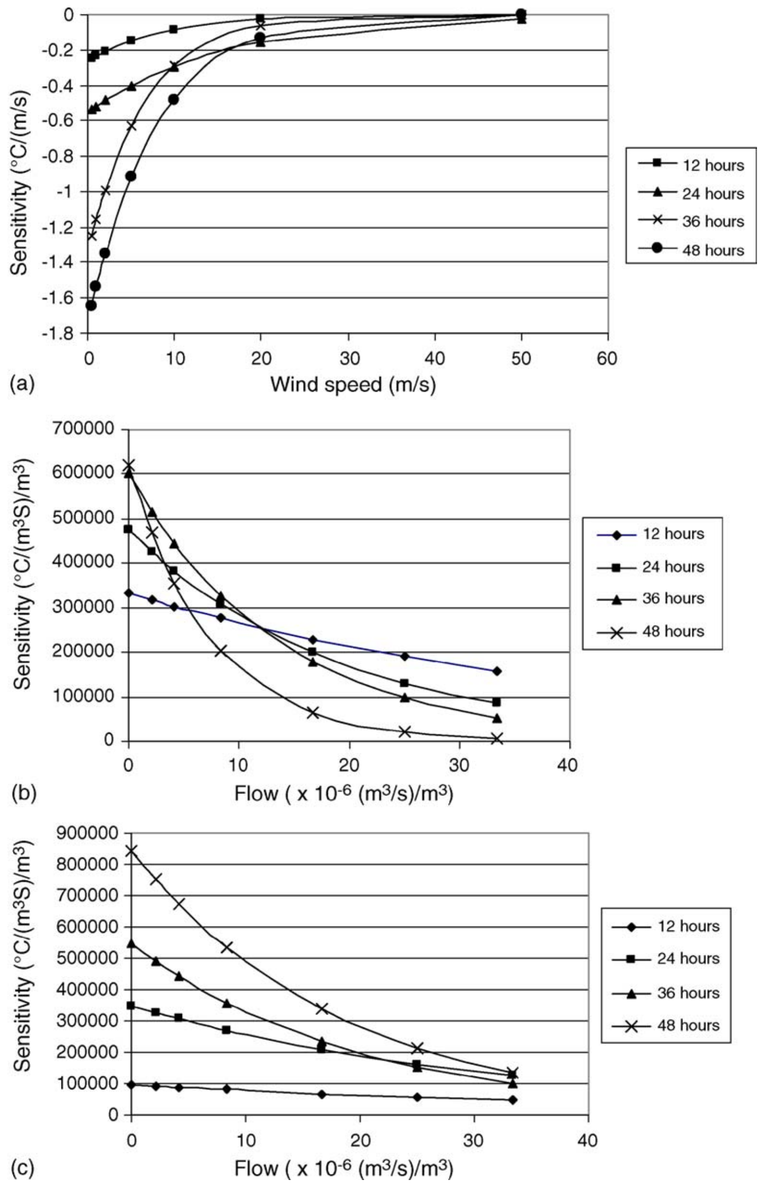


Fig. 4. The PHATR model's sensitivity to wind speed (a) and flow rate of warm (b) and cool (c) water varied exponentially. This means that in each case, the model's sensitivity was a function of wind speed and flow rate. For all three cases, the model's sensitivity decayed as wind speed or flow rate increased. For all curves, time was fixed at either 12, 24, 36 or 48 h. The model's sensitivity to variations in air temperature and solar radiation was not shown because sensitivity was constant for variations in these parameters for fixed times.

The model's sensitivity (the slope) as well as the intercept for each curve in Fig. 3b was calculated (Table 2). The model's sensitivity increased with time, from 0.0413 °C/% (12 h) to 0.1455 °C/% (48 h).

The "straightness" of the curves in Fig. 3b agreed with the analytical solution to Eq. (1), simplified and rewritten here:

$$\rho c_p V \frac{dT}{dt} = q_{solar} + \sum q \tag{5}$$

where $\sum q$ is the sum of all energy vectors except for solar radiation (W). Knowing that q_{solar} can be sub-

Table 1
PHATR's sensitivity to changes in average air temperature (the slope of the lines in Fig. 3a) was linear ($r > 0.999$ for all times) and increased with time linearly ($r > 0.999$) by 0.00695 °C/°C h

Time (h)	Sensitivity (°C_{output}/°C_{input})
12	0.097
24	0.190
36	0.271
48	0.348

stituted with the following equation:

$$q_{\text{solar}} = -1355\phi\cos\left(\frac{\pi t}{12}\right) \tag{6}$$

where ϕ is the percentage of solar radiation varied in the trial runs (%). Eq. (5) becomes:

$$\rho c_{\text{p}} V\frac{\mathrm{d}T}{\mathrm{d}t} = -1355\phi\cos\left(\frac{\pi t}{12}\right) + \sum q \tag{7}$$

Integrating Eq. (7) for $t = 0$ to some time t between 6 and 18 h gives:

$$T = T_i - \frac{16260\phi}{\pi\rho c_{\text{p}} V}\left(\sin\frac{\pi t}{12} - 1\right) + \int_0^t \left(\sum q\right)\mathrm{d}t \tag{8}$$

Variations in the model input (solar radiation, ϕ) caused variations in the difference in the model output (pond temperature). Making the input variation infinitely small (i.e. taking the first derivative of Eq. (8) with respect to ϕ) yielded:

$$\frac{\mathrm{d}T}{\mathrm{d}\phi} = -\frac{16260}{\pi\rho c_{\text{p}} V}\left(\sin\frac{\pi t}{12} - 1\right) \tag{9}$$

Table 2
Changes in PHATR's output (pond temperature) to changes in solar radiation (ϕ) can be calculated

Time (h)	Sensitivity (°C/%)	Intercept (°C)
12	0.0413	−4.13
24	0.0812	−8.08
36	0.1138	−11.32
48	0.1455	−14.41

Using the following equation ($r > 0.999$ for all times): $\Delta\text{output} = S\phi + \text{intercept}$, where S is the sensitivity and ϕ the percentage of the solar radiation used in the standard model run. These parameters are the linear regression parameters for the curves in Fig. 3b, the sensitivity being the slope of these lines.

Since time t is held constant, $\mathrm{d}T/\mathrm{d}\phi$ (the model's sensitivity with respect to radiation) is also constant. So the curve in Fig. 3b for $t = 12$ h must have a constant slope. If the time considered is 24 h, the corresponding curve in Fig. 3b should still have a constant slope, as is confirmed by the experimental data ($r = 0.999$). Using Eq. (7) again, but this time integrating from $t = 0$ to $t = 24$ h (or 6–18 h, since $q_{\text{solar}} = 0$ at night) yields:

$$T = T_i + \frac{32520\phi}{\rho c_{\text{p}} V\pi} + \int_0^{24} \left(\sum q\right) \tag{10}$$

Again, varying ϕ by very small increments yielded:

$$\frac{\mathrm{d}T}{\mathrm{d}\phi} = \frac{32520}{\rho c_{\text{p}} V\pi} \tag{11}$$

which is a constant. So the analytical solution and the experimental results in Fig. 3b agree. As a result, changes in solar radiation caused the pond temperature to change linearly, and in inverse dependence on the thermal mass (Eq. 11).

Variations in wind speed caused the pond temperature to vary exponentially ($r \geq 0.992$; Fig. 3c):

$$\Delta T = a(1 - \mathrm{e}^{-bv}) \tag{12}$$

where ΔT is the difference between the standard pond temperature and the trial pond temperature for a fixed time (°C), v the wind speed (m/s) and a and b are arbitrary statistical constants determined by curve expert (values in Table 3).

The derivative of Eq. (12) with respect to wind speed (and consequently, the equation for the model's sensitivity to wind speed) was

$$S = \frac{\mathrm{d}(\Delta T)}{\mathrm{d}v} = ab\,\mathrm{e}^{-bv} \tag{13}$$

For all times considered, the model's sensitivity with respect to wind speed decreased as the wind speed

Table 3
These parameters, used in Eqs. (12) and (13), were used to quantify the model's sensitivity to wind speed

Time (h)	a (°C)	b (s/m)	r
12	−2.310552	0.113607	0.999
24	−8.802606	0.062419	0.998
36	−8.763092	0.154033	0.999
48	−13.50179	0.129852	0.992

increased (Fig. 4a). Increasing the wind speed for all times beyond 20 m/s yielded changes in temperature less than 0.2 °C. After 48 h, the sensitivity ranged from −1.75 °C/(m/s) (for wind speed of 0 m/s) to 0 °C/(m/s) (for wind speed of 50 m/s). No relationship was sought between time and the model's sensitivity to wind speed, despite the fact that time does have an effect on sensitivity.

Changes in pond temperature varied exponentially to changes in the flow rate of warm water ($r > 0.999$ for all cases). The general form of the exponential equation is

$$\Delta T = a(1 - e^{-bQ}) \tag{14}$$

where Q is the flow rate ((m^3/s)/m^3). Its derivative is

$$S = \frac{d(\Delta T)}{dQ} = ab\,e^{-bQ} \tag{15}$$

The values of the statistical parameters a and b were calculated using Curve Expert (Table 4).

Increasing the flow rate of warm water into the pond decreased the model's sensitivity to this parameter, as described by the decaying exponential in Eq. (15) (Fig. 4b). After 48 h, the model's sensitivity ranged from 620,845 °C/(m^3/s)/m^3 (flow rate = 0 (m^3/s)/m^3) to 6972 °C/(m^3/s)/m^3 (flow rate = 33.3 × 10^{-6} (m^3/s)/m^3). Time did have an effect on the model's sensitivity to flow but no attempt was made to relate time to sensitivity since there was no flow between 6:00 and 18:00. The data did fit Eq. (12) with an $r > 0.999$.

The difference between the pond temperature for the trials and the standard increased as the flow of cold water increased (see Fig. 3e). As time progressed, the temperature difference between the trials and the standard also increased. Increasing the flow from 0 to 33.3 × 10^{-6} m^3/s increased the temperature difference between the standard and the trials by 2.3 °C after 12 h and 13.0 °C after 48 h. Changes in pond temperature varied exponentially to cold water flow (Eq. (14); $r > 0.999$; values for a and b shown in Table 5).

The model's sensitivity to cool water flow (Eq. (15), Fig. 4c) decreased as the flow rate increased. For instance, after 24 h, the sensitivity dropped from 348,712 to 125,682 °C/(m^3/s)/m^3. Changes in sensitivity were greatest after 48 h (708,805 °C/(m^3/s)/m^3) and smallest after 12 h (51,100 °C/(m^3/s)/m^3). Although time did have some impact on the model's sensitivity to flow, no relationship between time and sensitivity was sought. This is because Eq. (12), although empirically accurate in fitting the data in Figs. 3e and 4c, is not the proper simplified solution to Eq. (1). In other words, the curves describing the model's sensitivity to flow during the daytime do not reflect the fact that there was no cool water flow during the daytime.

An exponential curve was chosen to fit the plots 3c, 3d and 3e because of the presence of a similar term (particular solution) in the analytical solution to Eq. (1). For instance, when the flow rate of water was far more important than other energy transport mechanisms (Fig. 2), Eq. (1) was simplified:

$$\rho c_p V \frac{dT}{dt} = \rho c_p Q(T_{in} - T) \tag{16}$$

where T_{in}, a constant, is the temperature of the inlet warm water (°C). Solving this differential equation yielded:

$$T = T_{in}(1 - e^{-Qt/V}) + T_i\,e^{-Qt/V} \tag{17}$$

The first term in Eq. (17) is in the exact form of Eq. (14), which explains why the data was fitted in this way. However, because the nature of the second term

Table 4
The parameters (a, b) for Eqs. (14) and (15), describing the model's sensitivity to warm-water flow, were determined with curve expert (Hyams, 2001)

Time (h)	a (°C)	b (m^3/(m^3/s))
12	14.9	22 340
24	9.2	51 564
36	8.3	72 658
48	4.6	134 676

The correlation coefficient r, for all times, was 0.999.

Table 5
Parameters (a, b) used in Eqs. (14) and (15) determined the model's sensitivity to cold water flow

Time (h)	a (°C)	b (m^3/(m^3/s))
12	4.4	22 285
24	11.4	30 615
36	10.7	51 095
48	15.4	54781

The correlation coefficient r, for all times, was 0.999.

in Eq. (17) ($T_i\,e^{-Qt/V}$) is unknown and because Eq. (17) assumes that only the flow rate of water is important (which is not necessarily accurate), parameter a is not necessarily equal to T_{in} and b to t/V.

4. Discussion

For the aquaculturist, the sensitivity analysis is as an opportunity to determine which factors have a greater effect in controlling the pond temperature. By comparing the model's sensitivity to air temperature, solar radiation, wind speed and warm or cool water flow rates, factors which were important in controlling pond temperature should become obvious. However, there were problems with direct comparisons of the numerical values for sensitivity:

- Sensitivity was a function of time. Comparisons between the sensitivity to two different parameters, for example air temperature and wind speed, could only be done for a given time, since for all trials, as time changed, the sensitivity changed.
- The model's sensitivity to wind speed was a function of wind speed. Similarly, the model's sensitivity to flow rate was a function of flow rate. When comparing the model's sensitivity to these two parameters, the wind speed or flow rate should be specified.
- The units for various sensitivity measurements were different. Apples and oranges cannot be compared. Similarly, comparisons between sensitivity values with units °C/°C, °C/W, °C/(m/s) or °C/(m^3/s)/m^3 were not possible because each sensitivity value was measuring something different.

For these reasons, direct quantitative comparisons of sensitivity values were not possible. However, comparisons were possible if absolute variations in input were set to a specific value. It would be wrong to say that the model was more sensitive to changes in solar radiation as opposed to changes in air temperature. However, a comparison could be made between a change in air temperature of +5 °C for 12 h (absolute change in output = 0.417 °C) and a reduction of 25% of the maximum solar radiation for 12 h (absolute change in output = −1.078 °C). In this case, reducing the maximum solar radiation by 25% had a greater effect in changing the pond temperature. So, in this case, the model's sensitivity to changes in solar radiation was greater than the model's sensitivity to changes in air temperature.

By doing such comparisons for specific set parameters, it becomes possible to tell which parameters have a greater effect on the model's output, especially if those set parameters are set to extreme cases.

Changing the average air temperature did relatively little in affecting the pond temperature, when compared to variations in other input parameters. According to the theory, a change in the air temperature should directly affect convection because the temperature gradient between the pond and the air increases (the relative importance of convection was found to range between 0 and 27% of all energy transfer mechanisms; Lamoureux, 2003). Lowering the average air temperature to 0 °C after 48 h (absolute change in output = 6.3 °C) was not as effective as increasing the wind speed from 0 to 10 m/s (or 0–22.4 mph) (absolute change in output = 9.4 °C) or increasing the flow rate of cool 15 °C water from 0 to 16.7×10^{-6} (m^3/s)/m^3 (absolute change in output = 9.2 °C). Because the suggested variations in flow or wind speed were much more realistic than the suggested variations in air temperature for Baton Rouge, Louisiana, an aquaculturist from that area, trying to maintain a high pond temperature, should be more concerned about windy nights rather than cool nights.

Solar radiation, when left unbalanced, caused large increases in pond temperature. Over 48 h, the difference in pond temperature for the standard model run (100% solar radiation, pond temperature = 33.0 °C) and the trial run with 0% solar radiation (pond temperature = 18.5 °C) was 14.5 °C. A pond temperature of 18.5 °C in the spring/fall is more realistic than a pond temperature of 33.0 °C. However, the complete absence of solar radiation is itself unrealistic. Therefore, two observations can be drawn:

- Solar radiation was an important energy vector, since it can theoretically, when left unchecked cause the pond temperature to increase by 13.0 °C in 2 days. This increase was even more pronounced for the trial runs where the flow rate for the warm water

(36 °C) was varied (these trial runs had a fixed solar radiation of 100%, just like the standard model run). In this case, for a flow rate of 33.3×10^{-6} (m^3/s)/m^3, the pond temperature increased by 17.4 °C in 48 h.

- Solar radiation, for unheated ponds where the temperature is constant, had to be balanced, either by one or a combination of negative energy vectors. Such energy vectors included evaporation, where increasing the wind speed from 0 to 20 m/s decreased the standard pond temperature by 11.6 °C to a pond temperature of 21.3 °C after 48 h. This underlines: (1) the importance of evaporation and convection as prominent energy transport mechanism and (2) the wind speed as a controlling parameter for the pond temperature factor.

The sensitivity analysis, although insightful, failed to determine the model's sensitivity to interacting parameters. This is the main drawback of the ''one at a time'' type of sensitivity analysis (Saltelli, 2000). For instance, the combined effects of increasing wind speed while decreasing solar radiation would have caused the model's output to vary realistically.

This sensitivity analysis revealed useful information, and the model used for analyzing this information has good potential to help researchers to determine ways to save energy and run warmed systems more effectively. The sensitivity analysis revealed key areas of energy gains (primarily solar and hot water input) and losses (radiative losses, convective losses and warm water loss), with useful applications for practitioners. Further design and management implications are available from Lamoureux (2003).

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Appendix A

These weather variables described the standard weather used for the PHATR sensitivity analysis. For the equations used to describe solar radiation and air temperature, t = time in hours. Also note that between 18:00 and 6:00, solar radiation is equal to 0 W/m^2. Additional equations are in Lamoureux et al. (2006).

Weather variable	Value
Air temperature	$20 + 5\sin\left(\frac{\pi(t-8)}{12}\right)$
Relative humidity	90%
Solar radiation	$1355\sin\left(\frac{\pi(t-6)}{12}\right)$; 0
Wind speed	0 m/s

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